A Completely Invariant SUSY Transform of Supersymmetric QED

M.L. Walker

Abstract

We study the SUSY breaking of the covariant gauge-fixing term in SUSY QED and observe that this corresponds to a breaking of the Lorentz gauge condition by SUSY. Reasoning by analogy with SUSY's violation of the Wess-Zumino gauge, we argue that the SUSY transformation, already modified to preserve Wess-Zumino gauge, should be further modified by another gauge transformation which restores the Lorentz gauge condition. We derive this modification and use the resulting transformation to derive a Ward identitiy relating the photon and photino propagators without using ghost fields. Our transformation also fulfills the SUSY algebra, modulo terms that vanish in Lorentz gauge.

1 Introduction

Not only is supersymmetry (SUSY) believed necessary for the non-trivial unification of gravity and the gauge forces, but it is also expected to manifest in physics not much higher than the electroweak scale. One particular topic of current interest [1, 2] is the effect of the covariant gauge-fixing term in the component formalism. This term is not invariant to SUSY transformations in the component foralism and the non-linear corrections of Wess and Zumino do not restore it.

The conventional method of handling this situation is to introduce ghost fields and find SUSY BRST transformations [1, 2, 3]. Ghost fields are long known to be an essential part of non-Abelian field theories [4], and their use in SUSY theories for linearising the transformations and removing the auxiliary fields off mass-shell is also well-established [3]. The former of these objectives is considered necessary for the derivation of SUSY Slavnov-Taylory identities to relate the vertices of SUSY gauge theories [3], although the decoupling of the ghost fields in the Abelian case allows one to assume the invariance of the effective action even under the non-linear Wess-Zumino transformations and derive SUSY Ward identities [5]. The latter objective arises because, while it is understandably desirable to remove the auxiliary fields, to do so off-mass shell introduces errors that must be compensated. An alternative to this approach, in which the auxiliary fields are not removed but the scalar fields are instead "reinterpreted", has also been demonstrated [5] but is not in common use.

While we do not dispute these particular applications, we do dispute that the introduction of ghosts is necessary to obtain transformations that leave the entire SUSY QED action invariant. The conventional view seems to be that since the SUSY transformations found by Wess and Zumino in the seminal paper in which they constructed SUSY QED only apply to the gauge-invariant part of SUSY QED [6], then the gauge-fixing spoils, or at least limits, SUSY invariance. We argue instead that these transformations are incomplete, and then derive their completion. This turns out to be nothing more than the addition of another non-linear gauge transformation, similar to those found in Wess and Zumino's original paper.

A highly technical calculation by [1] finding the Noether current for the conventional SUSY BRST transformations in the component formalism, finds that SUSY is only a symmetry of the on-shell states, but not of the entire Fock space. In Lorentz gauge they found that SUSY is violated when the fields move off mass-shell by an amount proportional to

$$\delta_S(\partial \cdot A) = \bar{\zeta} \ \slashed{\partial} \lambda, \tag{1}$$

where A_μ, λ are the gauge and gaugino field respectively and ζ is the SUSY transform parameter. As the authors correctly assert, the discrepency arises from the SUSY breaking of the gauge-fixing term.

A later work [2] calculates the SUSY Ward identities for the Green's functions in SUSY gauge theories and finds that they do not hold at tree level, even in the Abelian case. The authors make the erroneous claim that the discrepency vanishes on mass-shell, and construct BRST identities whose corresponding Slavnov-Taylor identities do hold at tree level.

In this paper we go back to where Wess and Zumino left off and reexamine the SUSY breaking of the gauge-fixing term. In sec. 2 we derive the alteration to the conventional SUSY transformations needed to leave the covariant gauge-fixing term invariant also, and find that the resulting transformation obeys the SUSY algebra in Lorentz gauge. In sec. 3 we use our newfound transformation to derive a SUSY Ward identity relating the photon and photino propagators, and another relating the electron and selectron propagators. We conclude that the gauge-fixing term presents neither a fundamental difficulty, nor any unintuitive alterations to the Green's functions of SUSY QED. Instead, we find that SUSY Ward identities *can* be found, and that the claims of [1, 2] are artifacts of working with BRST identities based on incomplete SUSY transformations.

2 The Completely Invariant SUSY Transformations

As is well known [7], the matter fields in SUSY QED form a chiral multiplet

$$\begin{aligned}
\delta_S a &= -i\bar{\zeta}\psi \\
\delta_S b &= \bar{\zeta}\gamma_5\psi
\end{aligned}$$

$$\begin{aligned}
\delta_S\psi &= (f + i\gamma_5 g)\zeta + i\ \slashed{\partial}(a + i\gamma_5 b)\zeta \\
\delta_S f &= \bar{\zeta}\ \slashed{\partial}\psi \\
\delta_S g &= i\bar{\zeta}\gamma_5\ \slashed{\partial}\psi,
\end{aligned} \tag{2}$$

while the gauge field is part of a more general multiplet,

$$\begin{aligned}
\delta_S C &= \bar{\zeta}\gamma_5\chi \\
\delta_S \chi &= (M + i\gamma_5 N)\zeta + i\gamma^\mu(A_\mu + i\gamma_5\partial_\mu C)\zeta \\
\delta_S M &= \bar{\zeta}(\slashed{\partial}\chi + i\lambda) \\
\delta_S N &= i\bar{\zeta}\gamma_5(\slashed{\partial}\chi + i\lambda) \\
\delta_S A_\mu &= \bar{\zeta}\gamma_\mu\lambda - i\bar{\zeta}\partial_\mu\chi \\
\delta_S \lambda &= \frac{1}{2}(\gamma^\nu\gamma^\mu - \gamma^\mu\gamma^\nu)\partial_\mu A_\nu\zeta + i\gamma_5 D\zeta \\
\delta_S D &= i\bar{\zeta}\gamma_5\ \slashed{\partial}\lambda.
\end{aligned} \tag{3}$$

The elements C through to N are gauge degrees of freedom and it is well known that they are easily removed with a gauge transformation [6, 7]. The multiplets are combined in a gauge independant way to give the Lagrangian

$$\begin{aligned}
L &= |f|^2 + |g|^2 + |\partial_\mu a|^2 + |\partial_\mu b|^2 - \bar{\psi}\ \slashed{\partial}\psi \\[2mm]
&\quad - m(a^*f + af^* + b^*g + bg^* + i\bar{\psi}\psi) \\[2mm]
&\quad - ieA^\mu(a^*\overleftrightarrow{\partial}_\mu a + b^*\overleftrightarrow{\partial}_\mu b + \bar{\psi}\gamma_\mu\psi) \\[2mm]
&\quad - e[\bar{\lambda}(a^* + i\gamma_5 b^*)\psi - \bar{\psi}(a + i\gamma_5 b)\lambda] \\[2mm]
&\quad + ieD(a^*b - ab^*) + e^2 A_\mu A^\mu(|a|^2 + |b|^2) \\[2mm]
&\quad - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\bar{\lambda}\ \slashed{\partial}\lambda + \frac{1}{2}D^2 - \frac{1}{2\xi}(\partial_\mu A^\mu)^2,
\end{aligned} \tag{4}$$

where the gauge dependant superpartners of A_μ, λ and D have been gauged away (Wess-Zumino gauge).

As already noted, the gauge-fixing term spoils the SUSY invariance of the action. This is similar to the situation encountered by Wess and Zumino in their construction of the gauge invariant part of SUSY QED [6]. Rather than invoke ghost fields, they instead realised that the violation is due to the spoiling of Wess-Zumino gauge by SUSY tranformations. Their remedy was to follow the original SUSY transformation with a gauge transformation that restored their gauge.

It is the same here. The Lorentz gauge sets the longitudinal part of the gauge field to zero but the SUSY transformation contributes to the longitudinal

part so that it is no longer zero. The original SUSY transformation spoils the Lorentz gauge, just as it spoils the Wess-Zumino gauge. The remedy is the same: follow the SUSY transformation with a gauge transformation to restore the gauge. Such a transformation must exist. To find it, observe that we are looking for a gauge parameter θ such that

$$\partial^2 \theta_a = \delta_S(\partial \cdot A). \tag{5}$$

From this and eqn (1) it follows that

$$\theta_a = -\bar{\zeta}\frac{\slashed{\partial}}{\Box}\lambda. \tag{6}$$

The invariant SUSY transformation for SUSY gauge theories in component form and Lorentz gauge, or the Wess-Zumino-Lorentz gauge, is $\delta_{WZL} = \delta_S + \delta_{WZ} + \delta_L$ where δ_S is the original SUSY transformation given by eqs. (2,3) in Wess-Zumino gauge while δ_{WZ} is given by

$$
\begin{aligned}
\delta_{WZ}a &= 0, \\
\delta_{WZ}b &= 0, \\
\delta_{WZ}\psi &= -e\,\slashed{A}(a - i\gamma_5 b)\zeta, \\
\delta_{WZ}f &= -e\bar{\zeta}[a\lambda + ib\gamma_5\lambda - i\,\slashed{A}\psi], \\
\delta_{WZ}g &= -ei\bar{\zeta}[\gamma_5\lambda + ib\lambda + i\,\slashed{A}\gamma_5\psi], \\
\delta_{WZ}A_\mu &= 0, \\
\delta_{WZ}\lambda &= 0, \\
\delta_{WZ}D &= 0,
\end{aligned}
\tag{7}
$$

and δ_L by

$$
\begin{aligned}
\delta_L a &= i\bar{\zeta}\frac{\slashed{\partial}}{\Box}\lambda a, \\
\delta_L b &= i\bar{\zeta}\frac{\slashed{\partial}}{\Box}\lambda b, \\
\delta_L \psi &= i\bar{\zeta}\frac{\slashed{\partial}}{\Box}\lambda \psi, \\
\delta_L f &= i\bar{\zeta}\frac{\slashed{\partial}}{\Box}\lambda f, \\
\delta_L g &= i\bar{\zeta}\frac{\slashed{\partial}}{\Box}\lambda g, \\
\delta_L A_\mu &= \partial_\mu \bar{\zeta}\frac{\slashed{\partial}}{\Box}\lambda, \\
\delta_L \lambda &= 0, \\
\delta_L D &= 0,
\end{aligned}
\tag{8}
$$

That δ_{WZL} leaves the action invariant is obvious. The gauge-invariant part, constructed by Wess and Zumino [6], was shown by them to be invariant under

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both $\delta_S + \delta_{WZ}$ and any standard Abelian gauge transformation, including δ_L. The gauge-fixing term is unaffected by δ_{WZ} while θ was chosen so that $\delta_S + \delta_L$ would leave it invariant.

Less obvious, though not surprising, is that δ_{WZL} obeys the SUSY algebra in Lorentz gauge. For example,

$$[\delta_{WZL1}, \delta_{WZL2}]A_\mu = \bar{\zeta}_2 \; \slashed{\partial}\zeta_1 A_\mu + i\bar{\zeta}_2 \; \slashed{\partial}\zeta_1 \frac{\partial_\mu}{\Box}\partial \cdot A \tag{9}$$

where the final term vanishes in Lorentz gauge. Similarly for the electron field

$$[\delta_{WZL1}, \delta_{WZL2}]\psi = \bar{\zeta}_2 \; \slashed{\partial}\zeta_1 \psi - i\bar{\zeta}_2 \frac{\slashed{\partial}}{\Box}\zeta_1 \partial \cdot A\psi. \tag{10}$$

Similar results hold for the other fields.

3   SUSY Propagator Ward Identities

A powerful application of symmetries in quantum field theories is the derivation of Ward and Slavnov-Taylor identities relating the various Green's functions and proper vertices of a theory. Derivations of SUSY identities have had to work around the supposed SUSY violating properties of the gauge-fixing term, and the conventional approach is to replace the SUSY parameter with ghost fields [2, 3].

We calculate the SUSY Ward identity relating the photon and photino to be

$$
\begin{aligned}
0 &= \langle\delta(A_\mu(x)\lambda(y))\rangle \\
&= \langle A_\mu(x)A_\beta(y)\rangle_x \partial_\alpha \sigma^{\beta\alpha} - \langle\lambda(y)\bar{\lambda}(x)\rangle\gamma_\mu\zeta + \langle\lambda(y)\bar{\lambda}(x)\rangle\frac{x\partial_\mu}{x\slashed{\partial}}\zeta, \quad (11)
\end{aligned}
$$

where $\sigma^{\beta\alpha} = \frac{1}{2}(\gamma^\beta\gamma^\alpha - \gamma^\alpha\gamma^\beta)$. Note that the last term in this equation is due to δ_L, and is responsible for the failure of previous attempts [2] to derive this Ward identity. Eq. (11) holds in any gauge as the ξ-dependant part of the photon propagator is eliminated by multiplication with $\sigma^{\beta\alpha}$. In fact, eq. (11) relates the wave renormalisation of the photino to the vacuum polarisation according to

$$A_\lambda(p) = 1 + \Pi(p), \tag{12}$$

where the dressed photino propagator is given by $\langle\lambda(x)\bar{\lambda}(y)\rangle = \frac{-i}{A_\lambda(p)\slashed{p}}$ and the dressed photon propagator is $\langle A_\mu(x)\alpha A_\beta(y)\rangle = \frac{1}{p^2}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{1}{1+\Pi(p)} + \xi\frac{p_\mu p_\nu}{p^4}$. Indeed, eq. (12) is what one would naively expect [8].

We now investigate the identities relating the electron and selectron propagators. It is widely believed that since their wavefunction renormalisation is ξ dependant, the SUSY violation of the gauge-fixing term would cause the electron and selectron wavefunction renormalisations to differ, at least nonperturbatively. However our transformations are not violated by covariant gauge-fixing so such

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reasoning does not apply. The Ward identity relating the ψ and a propagators is

$$
\begin{aligned}
0 &= \langle \delta_{WZL}(\bar{\psi}(x)a(y)) \\
&= -i\bar{\zeta}\langle \psi(y)\bar{\psi}(x)\rangle + \bar{\zeta}\langle a(y)f^*(x)\rangle - i\bar{\zeta}_x \not{\partial}\langle a(y)a^*(x)\rangle,
\end{aligned}
\tag{13}
$$

as found originally by [9]. The non-linear contribution to this and all propagator Ward identities vanishes by the cluster decomposition principle [2, 10, 11]. This is an important result and one which should simplify further analyses.

4   Discussion

In the spirit of Wess and Zumino we have found a set of SUSY-based transformations that leave the action of SUSY QED completely invariant, essentially completing their work. Pleasingly, these transformations obey the SUSY algebra up to the Lorentz gauge-fixing condition, so they obey it completely in Lorentz gauge. While the derivation given is straightforward, the ramifications of this work are significant. The most important consequence is of course that an exact SUSY transformation of Abelian gauge-field theories *does* exist, even in Lorentz gauge. This should open the way for much simpler analysis of these theories. In particular, the necessity of ghost fields for deriving SUSY identities relating the Green's functions as claimed in some recent works [2] is seen to be false, as is the conventional belief (*eq.* [1]) that only gauge invariant terms can be supersymmetric. Our corrected SUSY transformation can used to derive exact Ward identities that relate the photon and photino, and we can rederive the original identities relating the propagators of the electron and selectrons, thus showing that they are not disrupted by the gauge-fixing term in spite of the wave-function renormalisation dependance on the gauge parameter.

The implications of this work run very deep. While the use of SUSY Slavnov-Taylor identities is common practice in mathematical analyses, SUSY Ward identities are still a stock tool, especially in lattice field theory [12]. It seems likely that their use can be broadened a great deal further, at least for Green's functions. Even if ghosts are used to linearise the SUSY transformations or remove the auxiliary fields, current BRST transformations are based on the transformations derived by Wess and Zumino [6], which are incomplete.

While we have worked here only in Lorentz gauge, the same general approach should be applicable in other component form gauge choices. The extension of this work to non-Abelian theories is very technically challenging, but necessary if this approach is to be applied to realistic theories.

Acknowledgments

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